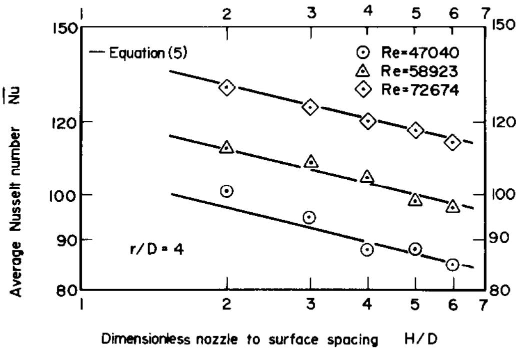

FIG. 4. Experimental results, $\overline{Nu}$ vs $H/D$.

ing empirical correlation is in the form,

$$\overline{Nu} = 0.216\,Re^{0.685}(H/D)^{-0.12}(r/D)^{-0.85}. \quad (5)$$

In Figs. 3 and 4, the predictions of equation (5) are given by the solid lines.

The experimental results obtained in the present work for unconstrained jets were compared with those available in the literature for unconstrained jets [1–7]. It was generally found that, for high $r/D$ values, lower heat transfer rates were obtained for constrained jets as compared to unconstrained jets for the same values of $Re$ and $H/D$. Superimposed on Fig. 3 is the prediction of equation (2) for unconstrained jets for $H/D$ equals 2 and $r/D$ equals 8, 4 and 2, respectively. These observations can easily be explained in terms of the resulting flow field near the heat transfer surface in each case.

For free impinging jets, the flow out of the nozzle will entrain air from the still surroundings forming a widening mixing region where an intensive momentum exchange will take place. As the flow approaches the stagnation region near the center of the heat transfer surface, it turns, forming what is essentially a wall jet separated from the surface by a boundary layer where the heat exchange with the wall takes place. For a constrained jet, the widening of the jet leaving the nozzle exit and its ability to entrain the surrounding air are limited by the backward flow leaving the surface. Accordingly, as the air reaches the surface its effective momentum and the width of the jet are somewhat lower than that of the free impinging jet. Also, the development of the boundary layer is slowed down by the presence of the constraining wall. Accordingly, boundary-layer stagnation and separation will take place near that wall. It is obvious that this change of the flow field will result in less heat removal from the surface. However, for low values of $r/D$ this effect diminishes. As shown in Fig. 3, for $r/D$ equals 2, equations (2) and (5) predict similar values of $Nu$.

## 4. CONCLUDING REMARKS

An experimental study was carried out to investigate the heat transfer characteristics of constrained air jets impinging on a flat surface. The results showed that, the heat transfer coefficients increased with increasing the jet Reynolds number $Re$ and with decreasing the radius of the heat transfer surface $r$ and the distance between the surface and the nozzle exit $H$. The data were successfully correlated by an empirical relationship, i.e. equation (5).

The results showed that, except for low values of $r/D$, constrained impinging jets are less capable of removing heat from flat surfaces compared to free, unconstrained, impinging jets under similar conditions. This was explained in terms of the resulting flow field in each case.

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# Analysis of interdiffusion in film absorption

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## 1. INTRODUCTION

THE COMBINED heat and mass transfer process taking place in film absorption has received growing attention in recent years and has been analyzed in several articles [1–6]. In the absorption process, encountered in numerous applications in chemical technology, the mass transfer is often accompanied by a significant heat effect, particularly when the absorbate is a vapor with a large latent heat. As was demonstrated in the above works, the heat and mass transfer processes are coupled in this case, and the concentration and temperature distributions are interdependent.

In all the above studies, the thermal effects due to diffusion [7] (also known as interdiffusion) were assumed to be negligible. Indeed, this assumption is quite realistic in most cases of practical importance. Yet, interdiffusion may become important for films with short exposure time [8] where large temperature and concentration gradients are present. Then, an additional term in the energy equation, expressing the energy transport due to the diffusive mass flux, may affect the solution.

This analysis considers the falling film model with the governing equations in their complete form, to evaluate the contribution of the interdiffusion. A short-exposure time solution is obtained by a similarity method.

## 2. MODEL AND EQUATIONS

Figure 1 describes schematically the system under consideration. A film of liquid solution, composed of substances

## NOMENCLATURE

| | |
|---|---|
| $C, C_0, C_e$ | concentration, initial concentration and equilibrium concentration of absorbate in solution [mol m$^{-3}$ solution] |
| $c_p$ | specific heat of liquid solution [J kg$^{-1}$ °C$^{-1}$] |
| $C_{Nu}$ | Nusselt number deviation factor, equation (24) [dimensionless] |
| $D, D^*$ | molecular, turbulent diffusion coefficient of absorbate in solution [m$^2$ s$^{-1}$] |
| $\bar{H}_a$ | heat of absorption of substance II in solution [J mol$^{-1}$] |
| $h_M$ | mass transfer coefficient from interface to bulk [m s$^{-1}$] |
| $h_T$ | heat transfer coefficient from interface to bulk [W m$^{-2}$ °C$^{-1}$] |
| $k, k^*$ | molecular, turbulent thermal conductivity of liquid solution [W m$^{-1}$ °C$^{-1}$] |
| $i$ | enthalpy of liquid solution [J kg$^{-1}$] |
| $Le$ | Lewis number, $D/\alpha$ [dimensionless] |
| $Nu_x$ | Nusselt nmber, $h_T x/k$ [dimensionless] |
| $Sh_x$ | Sherwood number, $h_M x/D$ [dimensionless] |
| $T, T_0, T_e$ | temperature, initial temperature and equilibrium temperature of solution [°C] |
| $u, u_0$ | flow velocity, velocity near interface [m s$^{-1}$] |
| $x$ | coordinate in direction of flow [m] |
| $y$ | coordinate in direction perpendicular to flow [m] |
| $Z$ | interdiffusion parameter, equation (11) [dimensionless]. |

Greek symbols

| | |
|---|---|
| $\alpha$ | thermal diffusivity of liquid solution, $k/\rho c_p$ [m$^2$ s$^{-1}$] |
| $\gamma$ | normalized concentration, equation (8) [dimensionless] |
| $\eta$ | similarity coordinate, $y/2\sqrt{xD/u_0}$ [dimensionless] |
| $\theta$ | normalized temperature, equation (8) [dimensionless] |
| $\lambda$ | normalized heat of absorption, $[D(C_e-C_0)\bar{H}_a]/[k(T_e-T_0)]$ [dimensionless] |
| $\rho$ | density of liquid [kg m$^{-3}$]. |

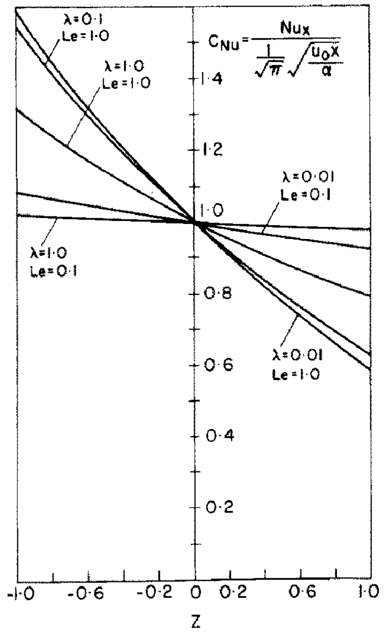

FIG. 1. Schematic description of a falling film. Typical temperature, concentration and velocity profiles are shown.

I (absorbent) and II (absorbate), flows down over an inclined surface. The film is in contact with stagnant vapor of substance II at constant pressure $P_v$. At $x = 0$, the liquid film is at a uniform temperature $T_0$, and composition $C_0$ corresponding to an equilibrium vapor pressure lower than $P_v$. This results in absorption taking place at the liquid–vapor interface, with the substance absorbed diffusing into the film; the heat of absorption produces a simultaneous heat transfer process. The flow may be laminar or turbulent.

In order to keep the treatment as general as possible, some of the assumptions made in earlier studies [5, 6] have been relaxed here. It is assumed that: (1) the film is thin and the flow is essentially one-dimensional; (2) no shear forces are exerted on the liquid by the vapor; (3) vapor pressure equilibrium exists between the vapor and liquid at the interface; and (4) heat transfer in the vapor phase is negligible compared to that in the liquid phase.

Under these assumptions, the simultaneous heat and mass transfer process in the system at steady-state is described by the diffusion and energy equations:

$$u\frac{\partial C}{\partial x}=\frac{\partial}{\partial y}\left(D^*\frac{\partial C}{\partial y}\right) \tag{1}$$

$$u\frac{\partial(\rho i)}{\partial x}=\frac{\partial}{\partial y}\left(k^*\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial y}\left[D^*\frac{\partial C}{\partial y}\frac{\partial(\rho i)}{\partial C}\right] \tag{2}$$

where diffusion and heat conduction in the $x$-direction have been neglected with respect to convection. Here $u$ is the steady-state component of the flow velocity, $D^*$ and $k^*$ are the generalized mass diffusivity and thermal conductivity, respectively, including, in the case of turbulent flow, the effect of turbulent eddies, and $i$ is the enthalpy of the solution. The second term on the RHS of the energy equation expresses the interdiffusion, namely, the flux of energy brought about by mass transfer.

The product $(\rho i)$ in equation (2) expresses the volumetric enthalpy (enthalpy per unit volume) of the film, and is a function of $T$ and $C$. By expressing the $x$ derivative of $(\rho i)$ on the LHS of (2) in terms of $x$-derivatives of $T$ and $C$, multiplying equation (1) by $[\partial(\rho i)/\partial C]_T$ and subtracting from (2), we obtain a simplified form of the energy equation:

$$u\left[\frac{\partial(\rho i)}{\partial T}\right]_C\frac{\partial T}{\partial x}=\frac{\partial}{\partial y}\left(k^*\frac{\partial T}{\partial y}\right)+D^*\frac{\partial C}{\partial y}\frac{\partial}{\partial y}\left[\frac{\partial(\rho i)}{\partial C}\right]_T. \tag{3}$$

For most liquids, the dependence of the enthalpy on temperature is almost linear over a significant range of the latter. Thus, the volumetric enthalpy may be expressed as:

$$(\rho i)=\rho c_p(T-T_0) \tag{4}$$

where $c_p$ is the specific heat and $T_0$ is chosen as the reference temperature at which the enthalpy is taken as zero. The density and specific heat are practically independent of temperature, but depend on concentration. Then, from (3):

$$\rho c_p u\frac{\partial T}{\partial x}=\frac{\partial}{\partial y}\left(k^*\frac{\partial T}{\partial y}\right)+D^*\frac{\partial C}{\partial y}\frac{\partial}{\partial y}\left[(T-T_0)\frac{\partial(\rho c_p)}{\partial C}\right]. \tag{5}$$

For most absorbent solutions the density and specific heat vary strongly with concentration, but the dependence of $\partial(\rho c_p)/\partial C$ on concentration is weak. It is therefore reasonable to assume this quantity to be constant with $y$, and equation (5) becomes:

$$u\frac{\partial T}{\partial x} = \frac{1}{\rho c_c}\frac{\partial}{\partial y}\left(k^*\frac{\partial T}{\partial y}\right) + \frac{D^*}{\rho c_p}\frac{\partial(\rho c_p)}{\partial C}\frac{\partial C}{\partial y}\frac{\partial T}{\partial y}. \tag{6}$$

It is convenient to introduce in the equations a dimensionless form of the temperature and concentration:

$$\theta = \frac{T-T_0}{T_e-T_0}; \quad \gamma = \frac{C-C_0}{C_e-C_0} \tag{7}$$

where $T_e$ is the equilibrium temperature of the solution at concentration $C_0$ with the vapor, and $C_e$ is the concentration of the solution at temperature $T_0$ in equilibrium with the vapor, both limiting values to those actually obtained in the simultaneous heat and mass transfer process [5, 6]. The diffusion and energy equations become :

$$u\frac{\partial \gamma}{\partial x} = \frac{\partial}{\partial y}\left(D^*\frac{\partial \gamma}{\partial y}\right) \tag{8}$$

$$u\frac{\partial \theta}{\partial x} = \frac{1}{\rho c_p}\frac{\partial}{\partial y}\left(k^*\frac{\partial \theta}{\partial y}\right) + D^*\left[\frac{(C_c-C_0)}{\rho c_p}\frac{\partial(\rho c_p)}{\partial C}\right]\frac{\partial \gamma}{\partial y}\frac{\partial \theta}{\partial y}. \tag{9}$$

Boundary conditions at the interface require vapor pressure equilibrium between the vapor and liquid phases, and a match between the heat and the mass fluxes. This depends on the thermodynamic property relations of the particular fluids at hand. For a linear absorbent [5, 6] having a linear temperature–concentration characteristic and a constant heat of absorption, these conditions become:

$$\left.\begin{array}{ll} \theta+\gamma = 1 & \text{(10a)} \\[2ex] \dfrac{\partial \theta}{\partial y} = \lambda\dfrac{\partial \gamma}{\partial y} & \text{(10b)} \end{array}\right\} \text{ at } y = 0$$

where $\lambda$ is a dimensionless heat of absorption.

## 3. SOLUTION

The term in square brackets on the RHS of equation (9):

$$Z = \left[\frac{(C_e-C_0)}{\rho c_p}\frac{\partial(\rho c_p)}{\partial C}\right] \tag{11}$$

is a dimensionless parameter describing a property of the film which reflects its sensitivity to interdiffusion. Other factors influencing the latter are the concentration and temperature gradients, as evident from equation (9). These gradients are particularly large at the entrance region of the film, where they are confined to a thin, developing boundary layer. In this study, we will therefore concentrate on a solution for the entrance region (short exposure time [8]). It will be shown that in this case an analytical similarity solution may be obtained.

At the thin boundary layer of the entrance region, the flow velocity is approximately uniform across the thickness; $u$ in equations (8) and (9) may be replaced by $u_0$—the velocity at the interface. Also, $k^*$ and $D^*$ become $k$ and $D$, respectively. The equations are to be solved with the interface boundary condition (10) and also:

$$\begin{array}{lll} T = T_0 & (\text{or } \theta = 0) & \left\{\begin{array}{l} \text{at } x = 0 \quad \text{(12a)} \\ \text{at } y \to \infty. \quad \text{(12b)} \end{array}\right. \\ C = C_0 & (\text{or } \gamma = 0) & \end{array}$$

Introducing the similarity variable $\eta = y/2\sqrt{xD/u_0}$, equations (8) and (9) may be rewritten as:

$$\gamma'' + 2\eta\gamma' = 0 \tag{13}$$

$$\frac{\theta''}{Le} + 2\eta\theta' + Z\theta'\gamma' = 0 \tag{14}$$

where a prime denotes differentiation with respect to $\eta$.

Equation (13) may be readily solved for $\gamma$. Integrating twice with respect to $\eta$ and applying the boundary condition (12b), we obtain:

$$\gamma = K_1[1-\operatorname{erf}(\eta)] \tag{15}$$

where $K_1$ is a constant of integration still to be determined. Similarly, equation (14) may be written in the form:

$$\frac{\mathrm{d}\theta'}{\theta'} = -Le(2\eta+Z\gamma')\,\mathrm{d}\eta \tag{16a}$$

yielding after integration:

$$\theta' = K_2\exp[-Le(\eta^2+Z\gamma)]. \tag{16b}$$

Applying the interface boundary condition (10b) ($\theta' = \lambda\gamma'$ at $\eta = 0$) makes it possible to evaluate $K_2$ in terms of $K_1$, and hence:

$$\theta' = -\frac{2\lambda K_1}{\sqrt{\pi}}\exp\{Le[ZK_1\operatorname{erf}(\eta)-\eta^2]\}. \tag{17}$$

Integration again yields an expression for $\theta$ with a new constant of integration, which may be calculated by applying the boundary condition (10a). Thus:

$$\theta = 1-K_1-\frac{2\lambda K_1}{\sqrt{\pi}}I(\eta) \tag{18}$$

where

$$I(\eta) = \int_0^\eta \exp\{Le[ZK_1\operatorname{erf}(t)-t^2]\}\,\mathrm{d}t \tag{19}$$

and the remaining constant $K_1$ may be determined from the boundary condition (12a):

$$\frac{\sqrt{\pi}}{2}\frac{(1-K_1)}{\lambda K_1} = \int_0^\infty \exp\{Le[ZK_1\operatorname{erf}(t)-t^2]\}\,\mathrm{d}t = I(\infty). \tag{20}$$

Thus, a complete solution for the distribution of the dimensionless concentration $\gamma$ and temperature $\theta$ is given by equations (15) and (18).

## 4. RESULTS AND DISCUSSION

The results of the solution show the temperature and concentration distributions in the film to depend on three dimensionless parameters: the Lewis number $Le$, the normalized heat of absorption $\lambda$, and the interdiffusion parameter $Z$. These three quantities affect the value of the constant $K_1$, which may be calculated from equation (20). The integral in this equation has been evaluated numerically. For the particular case where $Le = 1$, the integral (19) can be calculated analytically to give:

$$I(\eta) = \int_0^\eta \mathrm{e}^{ZK_1\operatorname{erf}(t)}\,\mathrm{e}^{-t^2}\,\mathrm{d}t = \frac{\sqrt{\pi}}{2ZK_1}[\mathrm{e}^{ZK_1\operatorname{erf}(\eta)}-1]. \tag{21}$$

It is also evident from equations (19) and (20) that for small $Z$ the dependence of $I(\eta)$ on $K_1$ and $Z$ is very weak. For $Z = 0$ it is easy to show that the solution reduces to that with no interdiffusion [5, 6].

The results for the temperature and concentration profiles make it possible to calculate the heat and mass transfer coefficients and hence the Nusselt and Sherwood numbers. Thus:

$$Sh_x = \frac{h_M x}{D} = \frac{x}{D}\frac{-D(\partial C/\partial y)_0}{C(0)-C_0} = \frac{1}{\sqrt{\pi}}\sqrt{\frac{u_0 x}{D}} \tag{22}$$

$$Nu_x = \frac{h_T x}{k} = \frac{x}{k}\frac{-k(\partial T/\partial y)_0}{T(0)-T_0} = \left[\frac{\lambda K_1}{\sqrt{Le}(1-K_1)}\right]\frac{1}{\sqrt{\pi}}\sqrt{\frac{u_0 x}{\alpha}}. \tag{23}$$

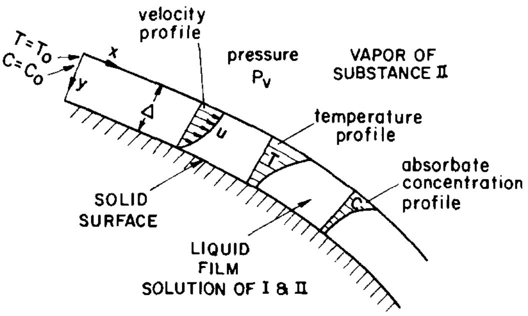

FIG. 2. Variation of $C_{Nu}$ with $Z$ for different $Le$ and $\lambda$.

It is evident that the Sherwood number is independent of $K_1$ and $Z$ and has the same value as in the case with no interdiffusion. The Nusselt number deviates from the $Z = 0$ case by a factor equal to the term in square brackets in equation (23). This deviation factor is defined as:

$$C_{Nu} = Nu_x \Big/ \left(\frac{1}{\sqrt{\pi}}\sqrt{\frac{u_0 x}{\alpha}}\right) = \frac{\lambda K_1}{\sqrt{Le}\,(1-K_1)}. \tag{24}$$

Figure 2 describes the variation of $C_{Nu}$ with $Z$ for several values of $Le$ and $\lambda$. Increasing $Z$ decreases the value of the Nusselt number, relative to its value in the absence of interdiffusion. It is also clear that the larger $\lambda$ and lower $Le$, the lower the value of $C_{Nu}$. It should be noted that larger $\lambda$ and lower $Le$ lead to a higher temperature at the vapor–liquid interface. Thus, while the heat transfer coefficient improves with decreasing $\lambda$ and increasing $Le$, the overall heat effect associated with the absorption process is reduced under this condition.

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# Thermal convection in a porous medium subject to transient heating and rotation

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## 1. INTRODUCTION

THE DESCRIPTION of thermal convection in a porous medium is mainly based on Darcy's law, which includes boundary and inertial effects [1]. The global effects of the fluid are derived by using volume average techniques [2–3]. Linear stability theory of the steady state may predict the onset of thermal convection at the marginal state [4], while nonlinear stability theory can differentiate the flow patterns and determine the subcritical instability [5–7]. Empirically, the cell patterns, first appearing at the marginal state, continue to manifest the same patterns in the weakly nonlinear stability state [8]. Foster [9] treats the thermal convection of transient state as an initial value problem. Amplification theory, requiring the empirically determined initial conditions, is applied to predict the critical time of thermal convection [10–13].

The onset of thermal convection of both steady and transient states in a porous medium rotating with an angular frequency is considered. Both upper and lower boundaries are free and fixed at a constant temperature $T_0$. The initial temperature distribution is nonlinear and is increased from the below at a constant rate $c$.

## 2. FORMULATION AND ANALYSIS

The dimensionless governing equations and conditions of the perturbed state, assuming the Boussinesq approximation, are

$$\left(\frac{\partial}{\partial t}+p_r\frac{\delta}{K}-p_r\nabla^2\right)^2\nabla^2 w = p_r\left(\frac{\partial}{\partial t}+p_r\frac{\delta}{K}-p_r\nabla^2\right)\nabla_1^2\theta - T_a D^2 w \tag{1}$$

$$\left(\frac{\partial}{\partial t}-\nabla^2\right)\theta = R(-DT)w \tag{2}$$

$$\left(\frac{\partial}{\partial t}-D^2\right)T = 0 \tag{3}$$

$$w = D^2 w = D^4 w = \theta = 0, \quad \text{at} \quad z = 0, 1 \tag{4}$$

$$T(0,t) = ct, \quad T(1,t) = 0 \quad \text{and} \quad T(z,0) = \frac{(1-z)z}{2}. \tag{5}$$

The solution for the basic temperature, from equations (3)